Conics - Ellipses

Why are they called conics?
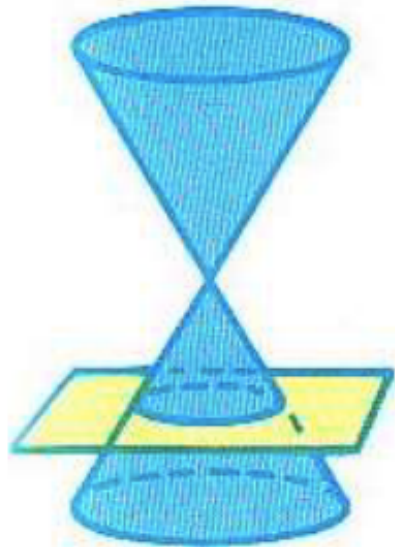
circle

parabola

ellipse

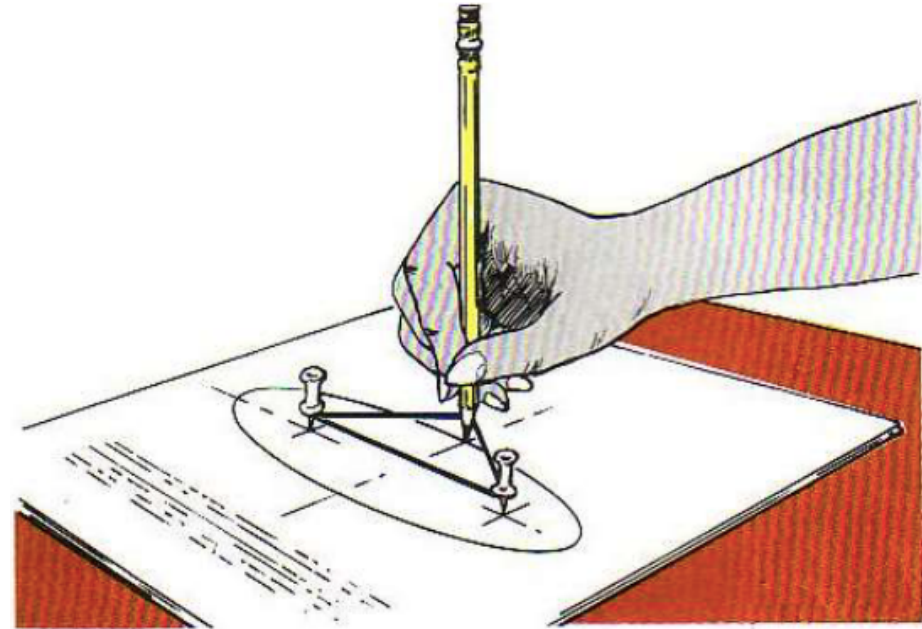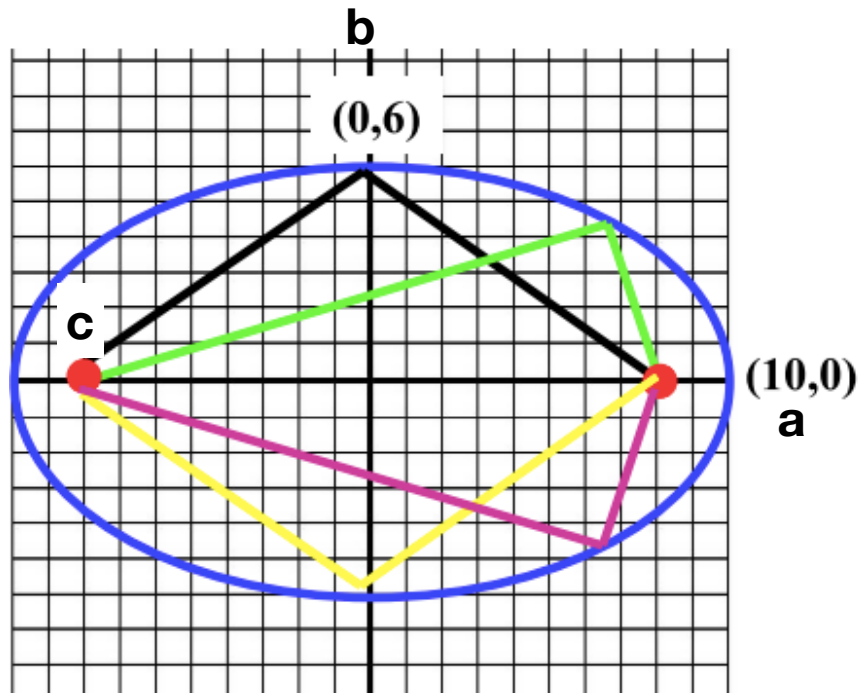hyperbola

Circle   *Parabola*   *Ellipse*   *Hyperbola*

What about a line?

Conics - Ellipses
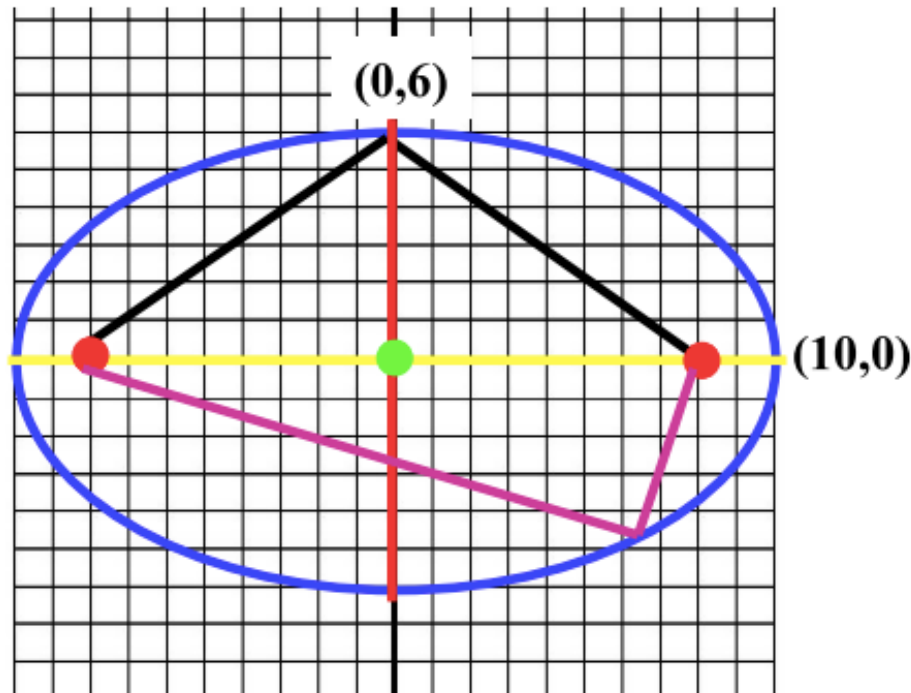
How do I draw an ellipse?

$$36x^2 + 100y^2 = 3600$$

$$\frac{x^2}{100} + \frac{y^2}{36} = 1$$

What is the distance from b to c?
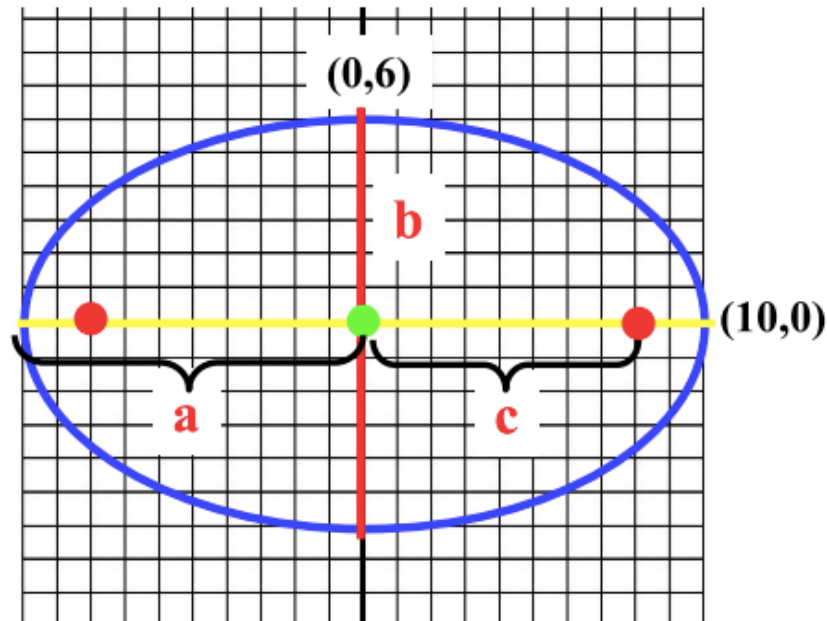
Conics - Ellipses

Vocabulary

- Foci - red points
- Center - green point
- Major Axis - yellow chord (long)
- Minor Axis - red chord (short)
- Focal Radii - length of black or purple lines

What is the focal radii of the ellipse?   20

Conics - Ellipses

What is the equation for an ellipse?

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{10^2} + \frac{y^2}{6^2} = 1$$

$$\frac{x^2}{100} + \frac{y^2}{36} = 1$$

What is the focal radius?   2a = 20

Conics - Ellipses

What is the equation for an ellipse?

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{10^2} + \frac{y^2}{6^2} = 1$$
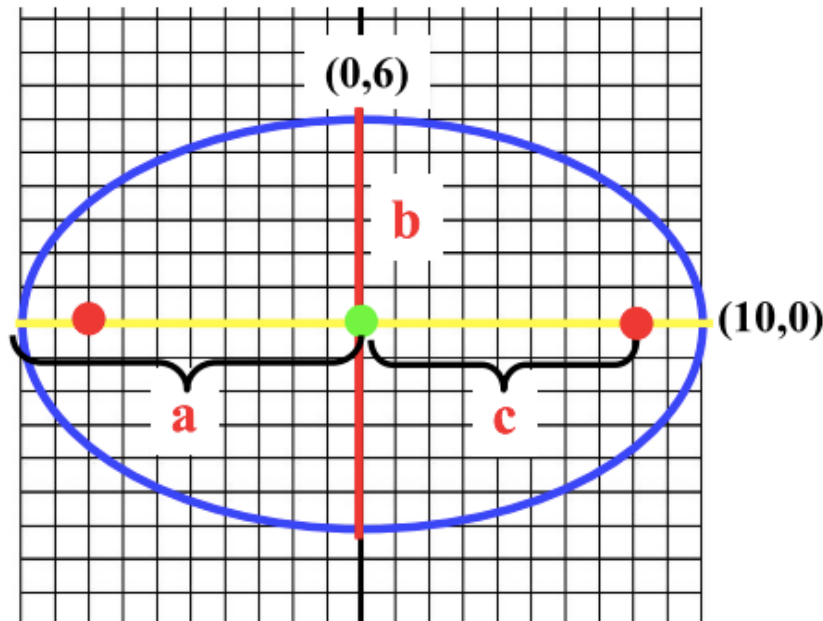
$$\frac{x^2}{100} + \frac{y^2}{36} = 1$$

(0,6)

b

(10,0)

a   **c**

What is the focal radius?   2a = 20

Write the equation and find focal radius (FR)

1) a=4, b=2

2) a=5, b=3

3) a=$\sqrt{26}$, b=$2\sqrt{5}$

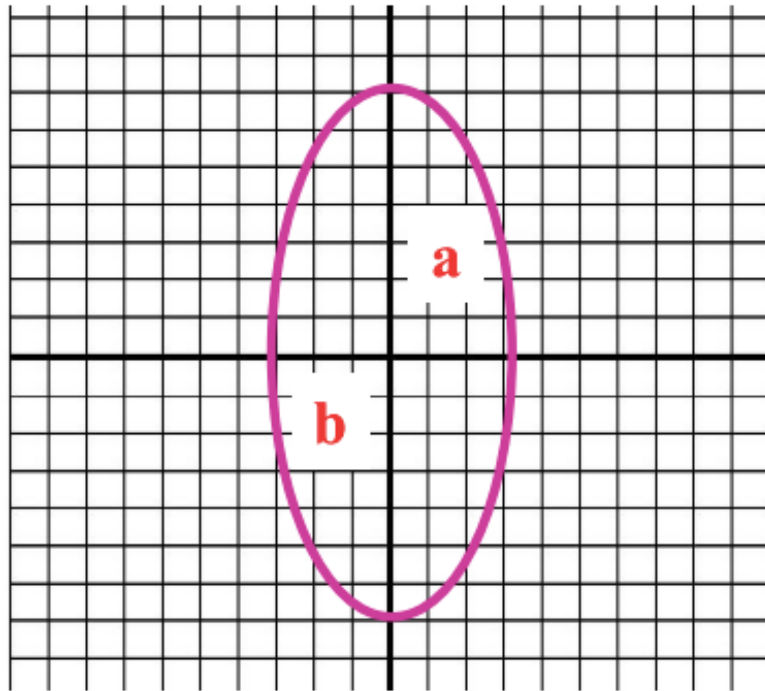$$\frac{x^2}{16} + \frac{y^2}{4} = 1; \; FR = 8$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1; \; FR = 10$$

$$\frac{x^2}{26} + \frac{y^2}{20} = 1; \; FR = 2\sqrt{26}$$

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What happens if we switch axes?

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\frac{x^2}{9} + \frac{y^2}{49} = 1$$

Conics - Ellipses

What if you were given the following equation.

$$4x^2 + y^2 = 64 \quad \longrightarrow \quad \frac{4x^2 + y^2 = 64}{64}$$

$$\frac{x^2}{16} + \frac{y^2}{64} = 1$$

What about $4x^2 + 4y^2 = 16$
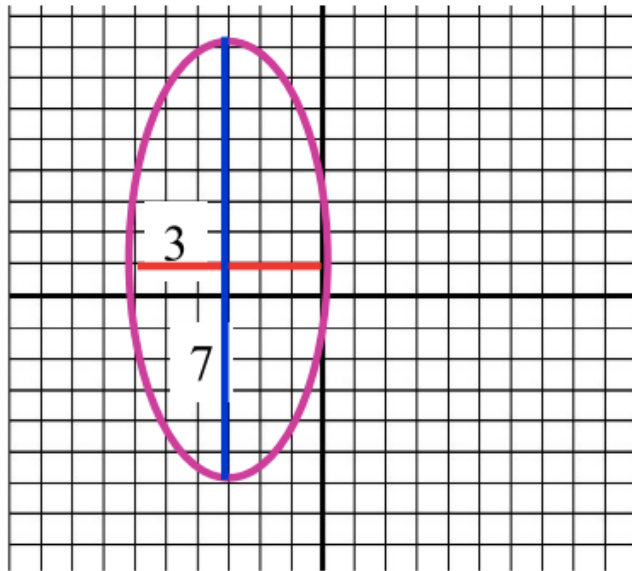

Write the equation.

1. $2x^2 + 3y^2 = 6$

$$\frac{x^2}{3} + \frac{y^2}{2} = 1$$

2. $12x^2 - 36 = -3y^2$

$$\frac{x^2}{3} + \frac{y^2}{12} = 1$$

Conics - Ellipses

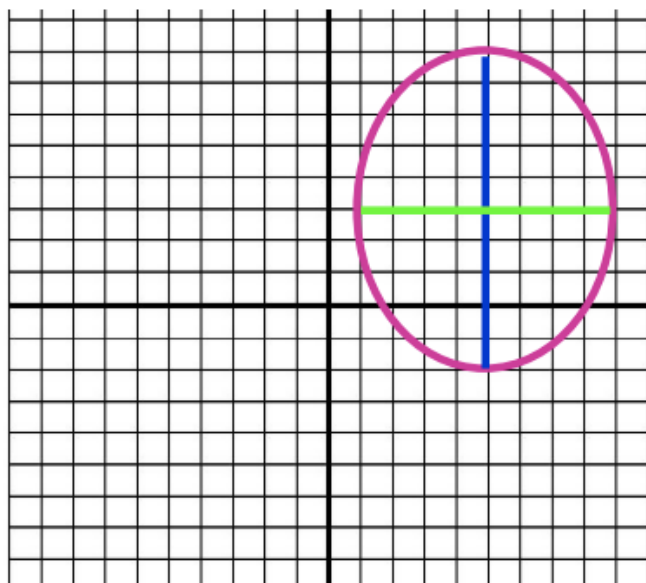What if the center was not at the origin? C(-3, 1)

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x-h)^2}{3^2} + \frac{(y-k)^2}{7^2} = 1$$

$$\frac{(x+3)^2}{9} + \frac{(y-1)^2}{49} = 1$$

Write the equation

$$\frac{(x-5)^2}{16} + \frac{(y-3)^2}{25} = 1$$

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$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$4x^2 + y^2 + 24x - 10y + 45 = 0$$

$$(4x^2 + 24x) + (y^2 - 10y) = -45$$

$$4(x^2 + 6x + ?) + (y^2 - 10y + ?) = -45 + ?$$

$$4(x^2 + 6x + 9) + (y^2 - 10y + 25) = -45 + 36 + 25$$

$$4(x+3)^2 + (y-5)^2 = 16$$

$$\frac{(x+3)^2}{4} + \frac{(y-5)^2}{16} = 1$$

Write the equation
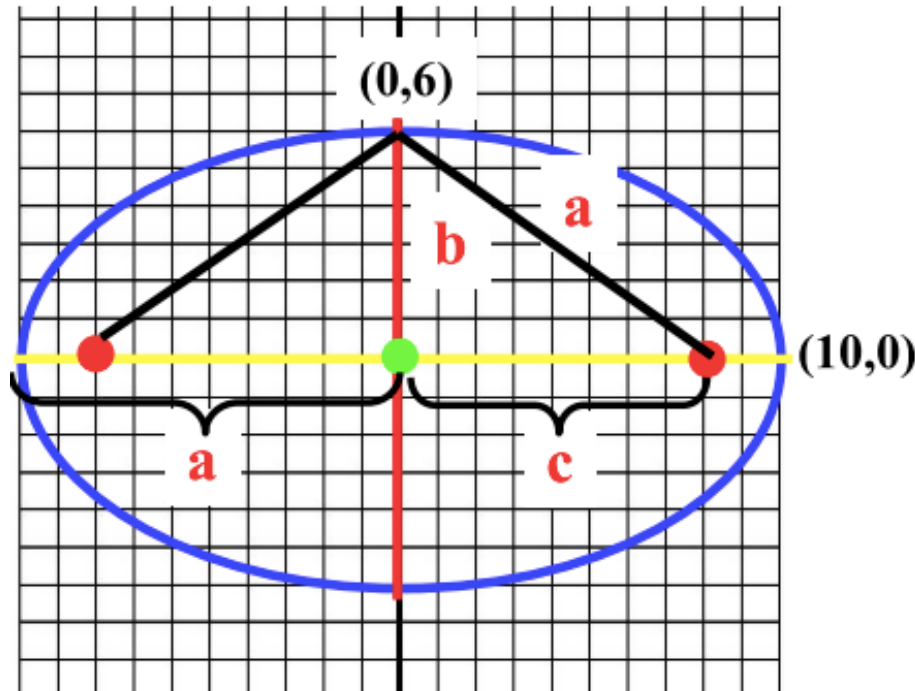
$$9x^2 + 4y^2 - 18x + 16y = 11 \qquad \frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$$

Conics - Ellipses

Is there a relationship between a, b, and c?

YES!! $\quad a^2 = b^2 + c^2$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

c is the distance from the center to each focus.

Conics - Ellipses

Graph and find the foci.

$$\frac{(x+3)^2}{4} + \frac{(y-5)^2}{16} = 1$$

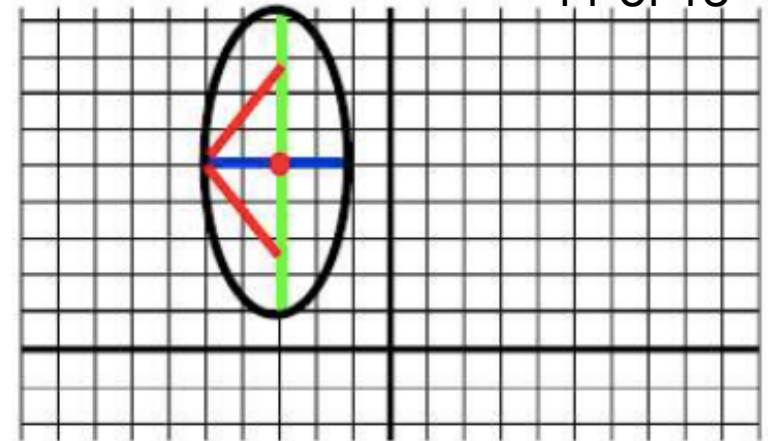Center (-3, 5) $a^2 = b^2 + c^2$

$a = 4$ $4^2 = 2^2 + c^2$

$b = 2$ $12 = c^2$

$\quad\quad\quad\quad\quad c = 2\sqrt{3}$

$f_1 = (-3, 5 - 2\sqrt{3})$ $f_2 = (-3, 5 + 2\sqrt{3})$

Conics - Ellipses

Graph and find the foci.

$$\frac{(x + 3)^2}{4} + \frac{(y - 5)^2}{16} = 1$$

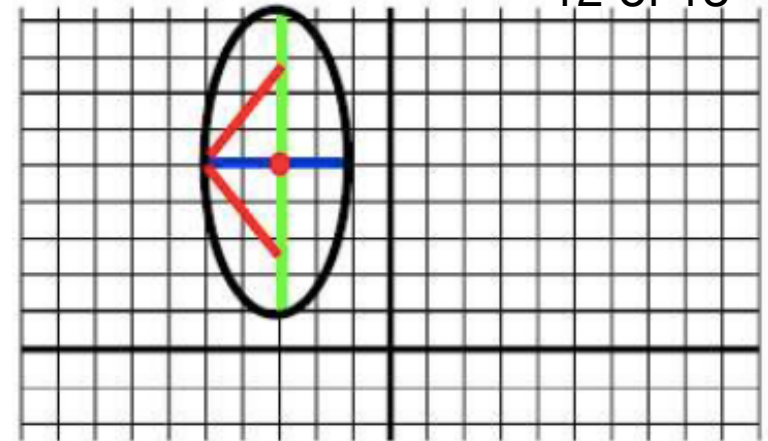Center (-3, 5) $a^2 = b^2 + c^2$

$a = 4$ $4^2 = 2^2 + c^2$

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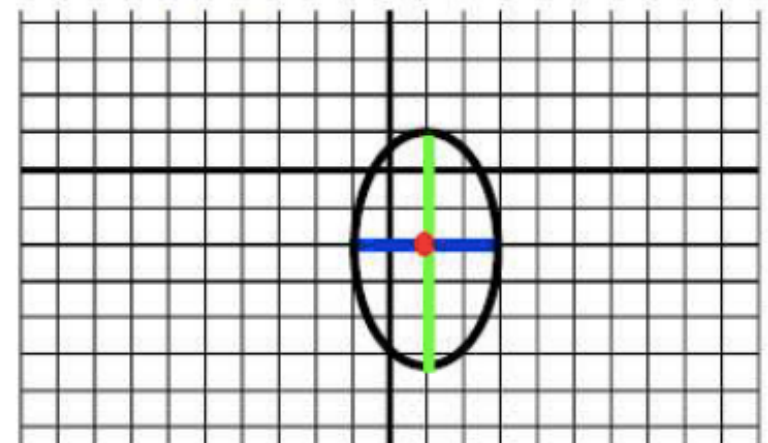$c = 2\sqrt{3}$ $f_1 = (-3, 5 - 2\sqrt{3})$ $f_2 = (-3, 5 + 2\sqrt{3})$

Graph and find the foci.

$$9x^2 + 4y^2 - 18x + 16y = 11$$

$$\frac{(x - 1)^2}{4} + \frac{(y + 2)^2}{9} = 1 \quad c = \sqrt{5}$$

$f_1 = (1, -2 + \sqrt{5})$ $f_2 = (1, -2 - \sqrt{5})$

Conics - Ellipses

1. Find the equation of an ellipse with the following characteristics

   vertices (3, 1), (7, 1)

   vertices (5, 5), (5, -3)

   $$\frac{(x-5)^2}{4} + \frac{(y-1)^2}{16} = 1$$

2. Find the equation of an ellipse with the following characteristics

   Focus (2, 2), (10, 2)

   vertices (0, 2), (12, 2)

   $$\frac{(x-6)^2}{36} + \frac{(y-2)^2}{20} = 1$$